

Vacuum stability in the $U(1)_\chi$ extended model with vanishing scalar potential at the Planck scale

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Abstract

We investigate the vacuum stability in a scale invariant local $U(1)_\chi$ model with vanishing scalar potential at the Planck scale. We find that it is impossible to realize the Higgs mass of 125 GeV while keeping the Higgs quartic coupling λ_H positive in all energy scales, that is, the same as the standard model. Once one allows $\lambda_H < 0$, the lower bounds of the Z' boson mass are obtained through the positive definiteness of the scalar mass squared eigenvalues, while the bounds are smaller than the LHC bounds. On the other hand, the upper bounds strongly depend on the number of relevant Majorana Yukawa couplings of the right-handed neutrinos N_ν . Considering decoupling effects of the Z' boson and the right-handed neutrinos, the condition of the singlet scalar quartic coupling $\lambda_\phi > 0$ gives the upper bound in the $N_\nu = 1$ case, while it does not constrain the $N_\nu = 2$ and 3 cases. In particular, we find that the Z' boson mass is tightly restricted for the $N_\nu = 1$ case as $M_{Z'} \lesssim 3.7 \text{ TeV}$.

1 Introduction

The standard model- (SM-)like Higgs boson was discovered at the LHC, and its mass was obtained by the ATLAS and CMS combined experiments as

$$M_h = 125.09 \pm 0.21 \text{ (stat.)} \pm 0.11 \text{ (syst.) GeV}, \quad (1)$$

with a relative uncertainty of 0.2% [1]. The SM predicts that the quartic coupling of the Higgs λ_H and its β function β_{λ_H} becomes zero below, but close to, the Planck scale ($M_{\text{Pl}} = 2.435 \times 10^{18} \text{ GeV}$) [2]. The negative quartic coupling causes a vacuum stability problem, which may suggest the appearance of new physics below the Planck scale. In fact, the vacuum of the Higgs potential is meta-stable in the SM, and the vacuum stability has been discussed in a number of works [3]–[19]. In particular, the multiple point principle (MPP) requires the vanishing λ_H and β_{λ_H} at a high energy scale, and it suggests a $135 \pm 9 \text{ GeV}$ Higgs mass with the top pole mass as $173 \pm 5 \text{ GeV}$ [20] (see also Refs. [21]–[27] for more recent analyses). Note that the conditions of the MPP could be naturally realized by the asymptotic safety of gravity [12].

The vanishing the Higgs quartic coupling near the Planck scale might suggest that the Higgs potential is completely flat at the Planck scale, and this possibility has been studied in Refs. [28]–[33]. In this context, the Higgs mass term is forbidden by a classical conformal invariance. The classical conformal invariance could be broken in general by radiative corrections via the Coleman-Weinberg (CW) mechanism [35], or a condensation in a strongly coupled sector like the QCD. In particular, in a flatland scenario, which is called in Ref. [30], an additional local $U(1)$ symmetry exists, and it is radiatively broken by the CW mechanism. Then, since the SM singlet scalar gets a nonzero vacuum expectation value (VEV), its mixing term with the Higgs becomes the Higgs mass term. If the mass term is negative, electroweak (EW) symmetry breaking could successfully occur. In Ref. [31], the authors investigated the possibilities of the flatland scenario in various $U(1)$ extended models.

In addition, the hierarchy problem for the Higgs mass can be solved in the flatland scenario as follows. From Bardeen’s argument [36], the quadratic divergence of the Higgs mass can always be multiplicatively subtracted at some energy scale. Once the mass term is renormalized at a high energy scale, e.g., the Planck scale, the quadratic divergence does not appear at lower energy scales. Then, the hierarchy problem is an issue only for logarithmic divergences. Since the renormalization group equation (RGE) of the Higgs mass term in the SM is proportional to itself, if it is zero at a high energy scale, it continues to be zero at lower energy scales as long as the theory is valid. However, if there is a mixing term between the Higgs and other scalar field, the RGE of the Higgs mass term includes a term proportional to the scalar mass squared. This term comes

from the logarithmic divergence due to the loop diagram of the scalar field. Then, the correction would be relevant for a realization of the Higgs mass when the scalar mass is not so large compared to the EW scale. Therefore, the hierarchy problem can be solved if no large intermediate scales exist between the EW and the Planck scales.

In this paper, we begin with a review of the flatland scenario in Sect. 2, in which we use the $U(1)_\chi$ extended model as in Ref. [33]. It is known that the CW mechanism can occur and the EW symmetry is successfully broken in this model (see Ref. [31]). However, a running of the singlet scalar quartic coupling is quite different from the typically expected one, when the number of relevant Majorana Yukawa couplings of the right-handed neutrinos is two, i.e., $N_\nu = 2$. Nevertheless, we find that the CW mechanism can also successfully occur in the $N_\nu = 2$ case. Next, we investigate the vacuum stability using two-loop RGEs in Sect. 3. We find that it is impossible to realize the Higgs mass of 125 GeV while keeping $\lambda_H > 0$ at all energy scales, that is, the same as the SM. Once one allows $\lambda_H < 0$, the lower bounds of the Z' boson mass are obtained through the positive definiteness of the scalar mass squared eigenvalues, while the bounds are smaller than the LHC bounds. On the other hand, the upper bounds strongly depend on N_ν . Considering the decoupling effects of the Z' boson and the right-handed neutrinos, the condition of the singlet scalar quartic coupling $\lambda_\phi > 0$ gives the upper bound in the $N_\nu = 1$ case, while it does not constrain the $N_\nu = 2$ and 3 cases. Finally, we mention the experimental bounds on the Z' boson mass in Sect. 4, and find that the Z' boson mass is tightly restricted for the $N_\nu = 1$ case to 2.24 (2.59) TeV $\lesssim M_{Z'} \lesssim 3.7$ TeV, where the lower bound corresponds to the ATLAS (CMS) result.

2 $U(1)_\chi$ extension of the SM in the flatland scenario

We consider the $U(1)_\chi$ extension of the SM, in which field contents are as shown in Table 1. A scalar potential is given by

$$V = \lambda_H |H|^4 + \lambda_\Phi |\Phi|^4 + \lambda_{\text{mix}} |H|^2 |\Phi|^2, \quad (2)$$

where H and Φ are a Higgs doublet and an SM singlet complex scalar, respectively. Since we assume the classical conformality, there are no dimensional parameters such as mass squared terms. In the flatland scenario, we impose that all the quartic couplings vanish at the Planck scale. The Lagrangian including right-handed neutrinos N is given by

$$\mathcal{L}_M = -Y_N^{\alpha i} \overline{L}_\alpha H N_i - Y_M^{ij} \Phi \overline{N}_i^c N_j + (h.c.), \quad (3)$$

where L is the lepton doublet, and α and i show the indices of the flavor and mass eigenstates, respectively. Since the type-I seesaw mechanism generates the active neutrino masses by integrating out right-handed neutrinos with TeV-scale masses, the Dirac

	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	$U(1)_\chi$
Q	$(3, 2, 1/6)$	$1/5$
U^c	$(\bar{3}, 1, -2/3)$	$1/5$
D^c	$(\bar{3}, 1, 1/3)$	$-3/5$
L	$(1, 2, -1/2)$	$-3/5$
E^c	$(1, 1, 1)$	$1/5$
N^c	$(1, 1, 0)$	1
H	$(1, 2, 1/2)$	$-2/5$
Φ	$(1, 1, 0)$	2

Table 1: Quantum numbers of the fields in the SM with $U(1)_\chi$ symmetry.

Yukawa couplings are typically $\mathcal{O}(10^{-6})$. Thus, we neglect Y_N for the RGE analyses in the following. Here, there are two $U(1)$ gauge bosons, and we take their kinetic terms as diagonal. Then, the covariant derivative is given by

$$D_\mu = \partial_\mu - ig_3 T^\alpha G_\mu^\alpha - ig_2 \tau^a W_\mu^a - ig_Y Y B_\mu^Y - i(g_{\text{mix}} Y + g_\chi X) B_\mu^X, \quad (4)$$

where Y and X denote $U(1)_Y$ and $U(1)_\chi$ charges, respectively. The $U(1)_\chi$ gauge boson is conventionally called the Z' boson, and we denote $Z'_\mu \equiv B_\mu^X$ hereafter (see Ref. [37] for a review of the Z' boson). The gauge couplings of $SU(3)_c$, $SU(2)_L$, $U(1)_Y$, and $U(1)_\chi$ are g_3 , g_2 , g_Y , and g_χ , respectively. In addition, there is a $U(1)$ mixing coupling g_{mix} , because it appears through loop corrections of fermions having both $U(1)_Y$ and $U(1)_\chi$ charges even if g_{mix} vanishes at some scale. In this paper, we impose $g_{\text{mix}}(M_{\text{Pl}}) = 0$, which would arise from breaking a simple unified gauge group into $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_\chi$. In particular, there is the well-known decomposition of the $SO(10)$ GUT as $SO(10) \rightarrow SU(5) \otimes U(1)_\chi$. Thus, when the $SO(10)$ GUT is broken at the Planck scale, $g_{\text{mix}}(M_{\text{Pl}}) = 0$ is naturally expected.

Let us explain the mechanism of the $U(1)_\chi$ symmetry breaking and the subsequent EW symmetry breaking. The $U(1)_\chi$ symmetry breaking is caused by the one-loop CW potential for the $U(1)_\chi$ sector, which is given by

$$V_\Phi(\phi) = \frac{1}{4} \lambda_\Phi \phi^4 + \frac{\phi^4}{64\pi^2} \left(10\lambda_\Phi^2 + 48g_\chi^4 - 8 \sum_{i=1}^3 y_{M_i}^4 \right) \left(\ln \frac{\phi^2}{M^2} - \frac{25}{6} \right), \quad (5)$$

around $\phi = M$ [35]. In this equation, we take $\Phi = \phi/\sqrt{2}$ in the unitary gauge, and Majorana Yukawa couplings of the right-handed neutrinos are diagonal as $Y_M^{ij} = y_{M_i} \delta_{ij}$. In our following analyses, we will take $\sum y_{M_i}^4 = N_\nu y_M^4$ for simplicity, where N_ν stands for the number of large Majorana Yukawa couplings that are enough to be effective in the RGE. Equation (5) satisfies the following renormalization conditions

$$\left. \frac{\partial^2 V_\Phi}{\partial \phi^2} \right|_{\phi=0} = 0, \quad \left. \frac{\partial^4 V_\Phi}{\partial \phi^4} \right|_{\phi=M} = 6\lambda_\Phi. \quad (6)$$

When the SM singlet scalar has a nonzero VEV $\langle\phi\rangle = v_\Phi$, we choose the renormalization scale at $M = v_\Phi$ to avoid the large log corrections, which have uncertainty in a large $\ln(\phi^2/v_\Phi^2)$ region. Then, the minimization condition of the potential (5) induces

$$\lambda_\Phi(v_\Phi) = \frac{11}{48\pi^2} (10\lambda_\Phi^2 + 48g_\chi^4 - 8N_\nu y_M^4) (v_\Phi). \quad (7)$$

When this relation is satisfied, the $U(1)_\chi$ symmetry is broken at v_Φ .

Once the SM singlet scalar gets a nonzero VEV v_Φ , the singlet scalar, the Z' boson, and the right-handed neutrinos become massive:

$$M_\phi = \sqrt{\frac{6}{11}}\lambda_\Phi(v_\Phi)v_\Phi, \quad M_{Z'} = 2g_\chi(v_\Phi)v_\Phi, \quad M_N = \sqrt{2}y_M(v_\Phi)v_\Phi, \quad (8)$$

respectively. To realize the CW mechanism successfully, the logarithmic terms of potential (5) should be effective compared to the first term. Thus, $\lambda_\Phi(v_\Phi)$ should be much smaller than $g_\chi(v_\Phi)$ and $y_M(v_\Phi)$, and the mass hierarchy $M_\phi \ll M_{Z'}, M_N$ is expected. As will be shown later, the typical value of M_ϕ is a few GeV, and then the singlet scalar does not decouple in the EW scale, while the Z' boson and the right-handed neutrinos decouple. From Eq. (7), the masses are approximately written as

$$M_\phi^2 \approx \beta_{\lambda_\Phi}(v_\Phi)v_\Phi^2 > 0, \quad \frac{M_{Z'}}{M_N} \approx \left(\frac{2N_\nu}{3}\right)^{1/4}. \quad (9)$$

Notice that $\beta_{\lambda_\Phi}(v_\Phi) > 0$ is required, since the scalar mass squared must be positive. On the other hand, $\beta_{\lambda_\Phi}(M_{\text{Pl}}) \leq 0$ must be satisfied to avoid $\lambda_\Phi < 0$ (which might cause the vacuum instability), since we impose $\lambda_\Phi(M_{\text{Pl}}) = 0$. Therefore, a running of λ_Φ is typically curved upward in the flatland scenario.

In general, a criterion for the successful CW mechanism has been derived as [31]

$$K = \frac{123x^2 - 50x + 12}{2 + N_\nu} \sqrt{\frac{N_\nu}{6}} < 1, \quad (10)$$

where x represents a generalized $B-L$ gauge charge: $x = 0$, $1/3$, and $x = 1/5$ correspond to $U(1)_R$, $U(1)_{B-L}$, and $U(1)_\chi$ models, respectively. In our case, i.e., for a $U(1)_\chi$ model,

$$K = 0.9417, 0.9988, 0.9786 \quad \text{for } N_\nu = 1, 2, 3, \quad (11)$$

respectively. Thus, in the $U(1)_\chi$ model, the flatland scenario can work for any $N_\nu = 1-3$. However, in the $U(1)_R$ and $U(1)_{B-L}$ models, the flatland scenario cannot work because of $K > 1$ for $N_\nu < 10$ and 20 , respectively. For $N_\nu = 0$, λ_Φ becomes negative in any energy scale below the Planck scale, because β_{λ_Φ} almost depends on the gauge quartic terms (see Eq. (34)). Thus, the flatland scenario cannot work in the $N_\nu = 0$ case.

Here, we comment on a running of λ_Φ in the $N_\nu = 2$ case, in which the value of K is almost equal to 1. It means that the terms $48g_\chi^4 - 8N_\nu y_M^4$ in β_{λ_Φ} , Eq. (34), are almost

vanishing. Then, two-loop order terms of β_{λ_Φ} are comparable to one-loop order terms, and β_{λ_Φ} becomes negative at all energy scales. Thus, the running of λ_Φ is monotonically and very slowly decreasing from the EW scale to the Planck scale [see Fig. 1-(b)], which is a quite different situation from that typically expected in the conventional flatland scenario. It is worth noting that the CW mechanism can also work in the $N_\nu = 2$ case, since the minimization condition (7) can be satisfied at the energy scale of v_Φ .

After the $U(1)_\chi$ symmetry breaking by the CW mechanism, the Higgs mass term is generated as

$$m_H^2(v_\Phi) = \frac{1}{2}\lambda_{\text{mix}}(v_\Phi)v_\Phi^2, \quad (12)$$

and the tree-level Higgs potential at v_Φ is given by

$$V_H(h) = \frac{1}{4}\lambda_H(v_\Phi)h^4 + \frac{1}{2}m_H^2(v_\Phi)h^2, \quad (13)$$

where we take $H = (0, (v_H + h)/\sqrt{2})^T$ in the unitary gauge. Below the energy scale of v_Φ , running of the Higgs mass term is governed by

$$\frac{dm_H^2}{d\ln\mu} = \frac{1}{16\pi^2} \left[m_H^2 \left(12\lambda_H + 6y_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 - \frac{3}{2} \left(g_{\text{mix}} - \frac{4}{5}g_\chi \right)^2 \right) + 2\lambda_{\text{mix}}M_\phi^2 \right]. \quad (14)$$

From Eq. (8), the last term in Eq. (14) is of the order of $\lambda_\Phi m_H^2$, and then it is negligible because of $\lambda_\Phi \ll 1$. In other words, $\lambda_{\text{mix}} \sim (v_H/v_\Phi)^2$ is required since m_H^2 is the EW scale, and then it is small enough to be neglected. Below $M_{Z'}$, the Z' boson decouples, and then the terms including g_{mix} and/or g_χ are omitted from Eq. (14). Note that the effects can be numerically neglected, since they are sufficiently small compared to other contributions in Eq. (14). As the VEV of the Higgs v_H , the minimization condition of the Higgs potential induces

$$v_H = \sqrt{\frac{-m_H^2(v_H)}{\lambda_H(v_H)}}, \quad (15)$$

where m_H^2 must be negative to realize the electroweak symmetry breaking. Notice that λ_{mix} , or m_H^2 , naturally becomes negative in the flatland scenario, since $\beta_{\lambda_{\text{mix}}}$ strongly depends on the gauge quartic terms which are always positive [see Eq. (35)]. Then, the Higgs pole mass is given by

$$M_h^2 = 2\lambda_H(v_H)v_H^2 + \Delta M_h^2, \quad (16)$$

where ΔM_h^2 is the Higgs self-energy correction to the Higgs pole mass [6]. The running of couplings controlled by the initial values of g_χ and y_M , and they are determined to realize $v_H \simeq 246$ GeV and $M_h \simeq 125$ GeV. On the other hand, once g_χ or y_M is fixed, the

other is uniquely determined by Eq. (7). Therefore, there is only one free parameter in the flatland scenario, and the physical quantities are uniquely predicted.¹

After the EW symmetry breaking, the singlet scalar and the Higgs are mixed by the λ_{mix} term. Then, the mass eigenvalues are different from M_ϕ and M_h . The scalar mass squared matrix is given by

$$\mathcal{M}^2 = \begin{pmatrix} M_h^2 & \frac{1}{2}\lambda_{\text{mix}}v_Hv_\Phi \\ \frac{1}{2}\lambda_{\text{mix}}v_Hv_\Phi & M_\phi^2 \end{pmatrix}, \quad (17)$$

where M_h and M_ϕ are given by Eqs. (16) and (8), respectively. Then, the scalar mixing angle θ is expressed as

$$\tan 2\theta = \frac{\lambda_{\text{mix}}v_Hv_\Phi}{M_h^2 - M_\phi^2}. \quad (18)$$

Since the flatland scenario expects $\lambda_\Phi \ll |\lambda_{\text{mix}}| \ll \lambda_H$ at a low energy scale, the lighter scalar mass squared eigenvalue is approximately written by

$$M_{\phi'}^2 \approx M_\phi^2 - \frac{\lambda_{\text{mix}}^2 v_H^2 v_\Phi^2}{4(M_h^2 - M_\phi^2)}. \quad (19)$$

It would be negative for a large $|\lambda_{\text{mix}}|$. We will discuss the positive definiteness of the scalar mass squared eigenvalues in the next section.

In the same way, the $U(1)$ gauge bosons are mixed by the g_{mix} term. It is potentially dangerous, because the ρ -parameter deviates from unity at the tree level. The mass term of the Z and Z' bosons are given by

$$\mathcal{L}_Z = \frac{1}{2}(Z_\mu, Z'_\mu)M_{ZZ'}^2 \begin{pmatrix} Z^\mu \\ Z'^\mu \end{pmatrix}, \quad M_{ZZ'}^2 = \begin{pmatrix} M_Z^2 & \delta M^2 \\ \delta M^2 & M_{Z'}^2 + \frac{1}{4}(g_{\text{mix}} - \frac{4}{5}g_\chi)^2 v_H^2 \end{pmatrix}, \quad (20)$$

where M_Z is the SM one as $M_Z^2 = (g_Y^2 + g_2^2)v_H^2/4$, and the second term of Z' boson mass is obtained by the Higgs VEV after the EW symmetry breaking, which is much smaller than $M_{Z'}^2$, because of $v_H \ll v_\Phi$. The mixing term is given by

$$\delta M^2 = \frac{1}{4}\sqrt{g_Y^2 + g_2^2} \left(g_{\text{mix}} - \frac{4}{5}g_\chi \right) v_H^2, \quad (21)$$

and the mass matrix is diagonalized by

$$\tan 2\theta_Z = \frac{2\delta M^2}{M_Z^2 - \left(M_{Z'}^2 + \frac{1}{4}(g_{\text{mix}} - \frac{4}{5}g_\chi)^2 v_H^2 \right)}. \quad (22)$$

¹ More accurately, there are more degrees of freedom for the Majorana Yukawa coupling matrix Y_M^{ij} . But, we had taken $\text{tr}[Y_M^{ij}] = N_\nu y_M$ for simplicity, and analyze independently by fixing $N_\nu = 1, 2$, and 3.

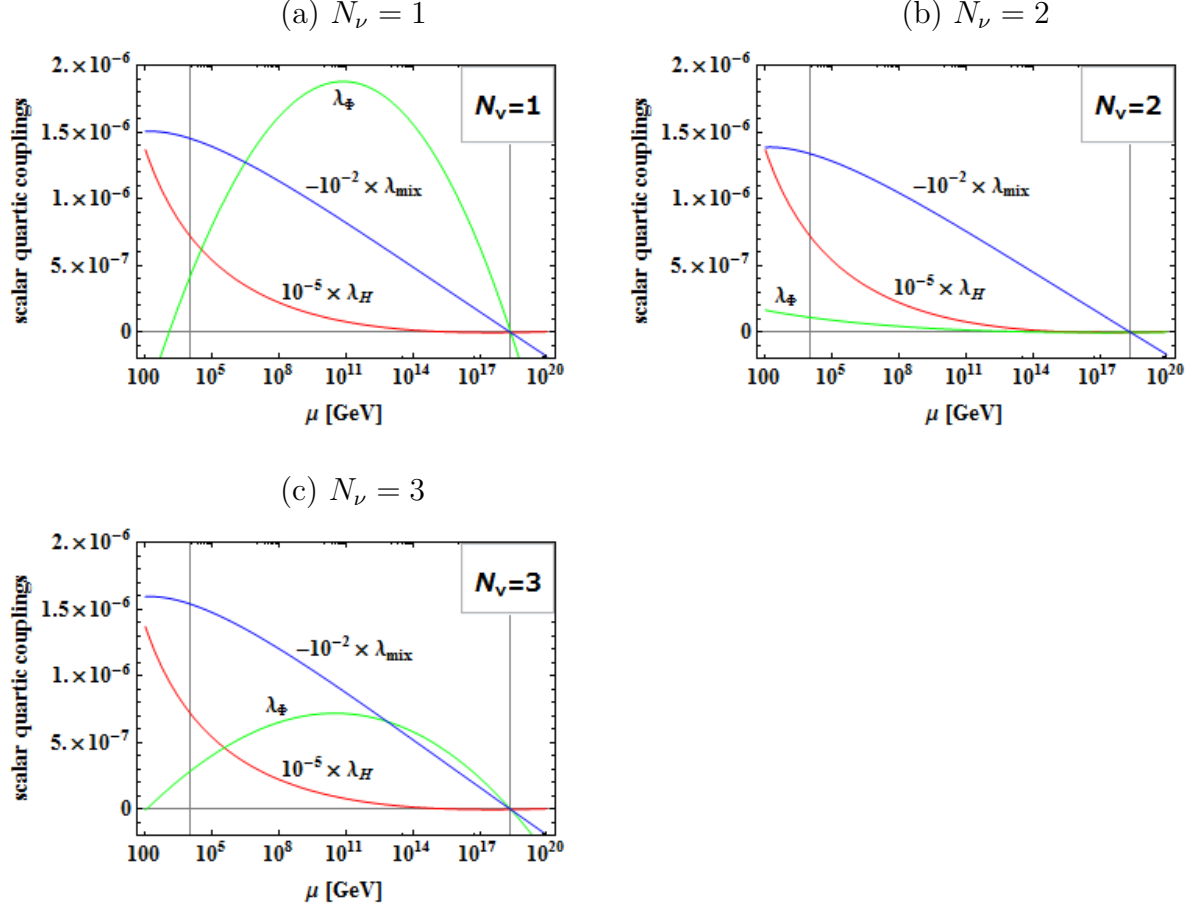


Figure 1: Example of runnings of quartic couplings for $N_\nu = 1, 2$ and 3 . The red, green, and blue lines correspond to $10^{-5} \times \lambda_H$, λ_Φ , and $-10^{-2} \times \lambda_{\text{mix}}$, respectively. Two vertical grid lines represent v_Φ and M_{Pl} , respectively. The decoupling effects of the Z' boson and the right-handed neutrinos are not considered in these figures.

After diagonalizing the mass matrix, the lighter mass squared eigenvalue is approximately obtained by

$$M_1^2 \approx M_Z^2 - \frac{(\delta M^2)^2}{\left(M_{Z'}^2 + \frac{1}{4} \left(g_{\text{mix}} - \frac{4}{5}g_\chi\right)^2 v_H^2\right) - M_Z^2}, \quad (23)$$

which is smaller than M_Z^2 . The ρ -parameter deviates from unity when M_1 is different from M_Z . We will also discuss the deviation of the ρ -parameter in the next section.

3 Constraints by the vacuum stability

In Fig. 1, we show runnings of the scalar quartic couplings. The $N_\nu = 1$ and 3 cases show the behavior as expected in the conventional flatland scenario, that is, a running of λ_Φ is curved upward, and λ_{mix} is negative to realize the negative Higgs mass term. On the

other hand, for $N_\nu = 2$, a running of λ_Φ behaves quite differently from $N_\nu = 1$ and 3. The running of λ_Φ is monotonically and slowly decreasing from the EW scale to the Planck scale as mentioned above. In all $N_\nu = 1-3$, the Higgs mass of 125.1 GeV is realized with the top pole mass of 171 GeV, and the $U(1)_\chi$ is broken at $v_\Phi \simeq 10$ TeV. In these cases, the singlet scalar, the Z' boson, and right-handed neutrino are massive for $N_\nu = 1, 2, 3$ as $M_\Phi \simeq 5.0$ GeV, 2.7 GeV, 3.9 GeV, $M_{Z'} \simeq 2.0$ TeV, 2.0 TeV, 2.0 TeV, and $M_N \simeq 2.3$ TeV, 1.9 TeV, 1.7 TeV, respectively. The values of the ratio $M_{Z'}/M_N$ agree very well with the predicted values from Eq. (9).

We investigate the parameter spaces allowed by the vacuum stability using two-loop RGEs. Since there is a few percent error for a running of the Higgs quartic coupling λ_H in one-loop RGEs, we have to use two-loop RGEs for a discussion of the vacuum stability. Adding the singlet scalar into the SM, the vacuum stability conditions are given by [38]

$$\lambda_H > 0, \quad \lambda_\Phi > 0, \quad 4\lambda_H\lambda_\Phi - \lambda_{\text{mix}}^2 > 0. \quad (24)$$

These conditions should be satisfied in any energy scale. If all the quartic couplings are positive, the potential is trivially bounded from below, and the vacuum is stable. The last condition in Eq. (24) shows the upper bound of $|\lambda_{\text{mix}}|$. Note that there are the non-trivial vacuum stability conditions of $\lambda_{\text{mix}} < 0$.

For our analyses, we take g_χ as a free parameter, and show its dependences on the other physical quantities in Fig. 2. Since $M_{Z'}$ and M_N satisfy Eq. (9), they are almost the same value. Although this figure shows the result for $N_\nu = 1$, the predicted physical quantities are almost the same for $N_\nu = 2$ and 3. This is because the runnings of the couplings, except λ_Φ , are almost the same for any N_ν . The left and right shaded regions correspond to constraints obtained by the vacuum stability conditions and the positive definiteness of the scalar mass squared eigenvalues, respectively. We will explain the constraints while discussing each condition below.

First, we consider the Higgs quartic coupling λ_H . To realize $\lambda_H > 0$ in any energy scale, the β function of λ_H at the Planck scale should satisfy $\beta_{\lambda_H}(M_{\text{Pl}}) \leq 0$ because of $\lambda_H(M_{\text{Pl}}) = 0$. In the SM, once $\lambda_H(M_{\text{Pl}}) = 0$ and $\beta_{\lambda_H}(M_{\text{Pl}}) \leq 0$ is imposed, we can find $M_t \gtrsim 173$ GeV and $M_h \gtrsim 129$ GeV [2, 19], while this lower bound of the Higgs mass is disfavored by the experiments. In the flatland scenario, $\beta_{\lambda_H}(M_{\text{Pl}})$ is given by

$$\beta_{\lambda_H}(M_{\text{Pl}}) = \frac{1}{(4\pi)^2} \left[-6y_t^4 + \frac{3}{8} \left\{ 2g_2^4 + \left(g_2^2 + g_Y^2 + \frac{16}{25}g_\chi^2 \right)^2 \right\} \right], \quad (25)$$

up to the one-loop level. The larger g_χ becomes, the larger the top Yukawa coupling y_t (or the top pole mass M_t) becomes compared with the SM in order to realize the 125 GeV Higgs mass. The left figure of Fig. 3 shows the relation between M_t and $\beta_{\lambda_H}(M_{\text{Pl}})$, in which the dots realize the Higgs mass in the range of Eq. (1). Then, the larger M_t becomes,

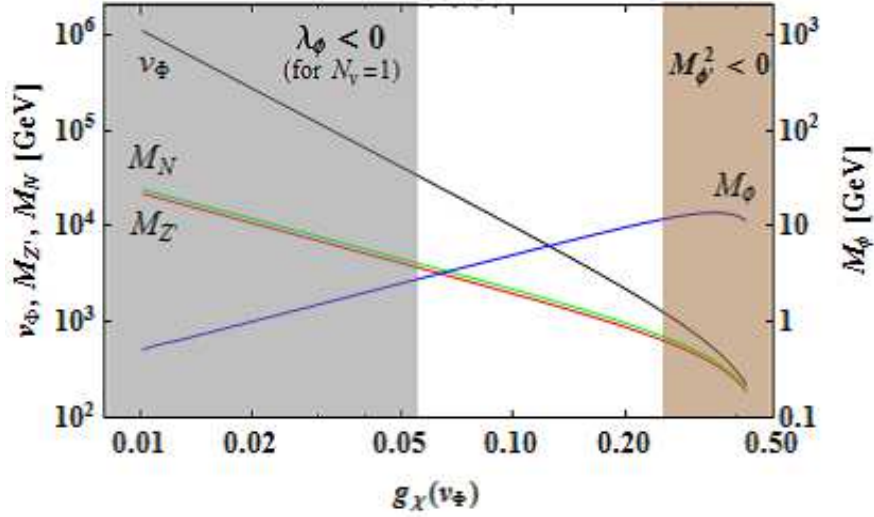


Figure 2: $U(1)_\chi$ gauge coupling dependences on the singlet scalar VEV and new particle masses obtained by Eq. (8). The left and right shaded regions are excluded by the $\lambda_\Phi < 0$ and $M_\Phi^2 < 0$ conditions, respectively. This figure shows the $N_\nu = 1$ case, and the left shaded region does not appear in the $N_\nu = 2$ and 3 cases.

the larger $\beta_{\lambda_H}(M_{\text{Pl}})$ becomes, while the Higgs mass cannot be realized by $M_t \lesssim 171 \text{ GeV}$. We find that it is impossible to simultaneously realize both $\beta_{\lambda_H}(M_{\text{Pl}}) \leq 0$ (or $\lambda_H > 0$) and $M_h \simeq 125 \text{ GeV}$.

On the other hand, once one gives up $\lambda_H > 0$ in any energy scale and imposes $\lambda_H(M_{\text{Pl}}) = 0$, the measured Higgs mass as $M_h \simeq 125 \text{ GeV}$ can be realized by $M_t \simeq 171 \text{ GeV}$ in the SM. Although λ_H becomes negative below the Planck scale, the vacuum is meta-stable, which is phenomenologically allowed. The same thing can be said in the flatland scenario unless the running of λ_H does not drastically change from that in the SM. As g_χ becomes larger, $M_h \simeq 125 \text{ GeV}$ can be realized by the larger M_t compared to the SM case, which is shown in the right figure of Fig. 3. When we allow $\lambda_H < 0$ as long as the vacuum is meta-stable, $M_h \simeq 125 \text{ GeV}$ can be realized by $g_\chi \simeq 0.4$ corresponding to the experimentally favored value, $M_t \simeq 173 \text{ GeV}$ [39]. However, the large g_χ region as $g_\chi \gtrsim 0.2$ is excluded for $N_\nu = 1$ by the positive definiteness of the scalar mass squared eigenvalues, as mentioned below.

Next, we consider the singlet scalar quartic coupling λ_Φ . In Fig. 1 (a), λ_Φ seems to become negative an order of magnitude below the singlet scalar VEV v_Φ . However, in fact, we can find $\lambda_\Phi > 0$ is realized as follows. After the $U(1)_\chi$ symmetry breaking, the Z' boson and the right-handed neutrinos become massive. Since their masses are the same order of magnitude as v_Φ , they would decouple and be integrated out from the theory

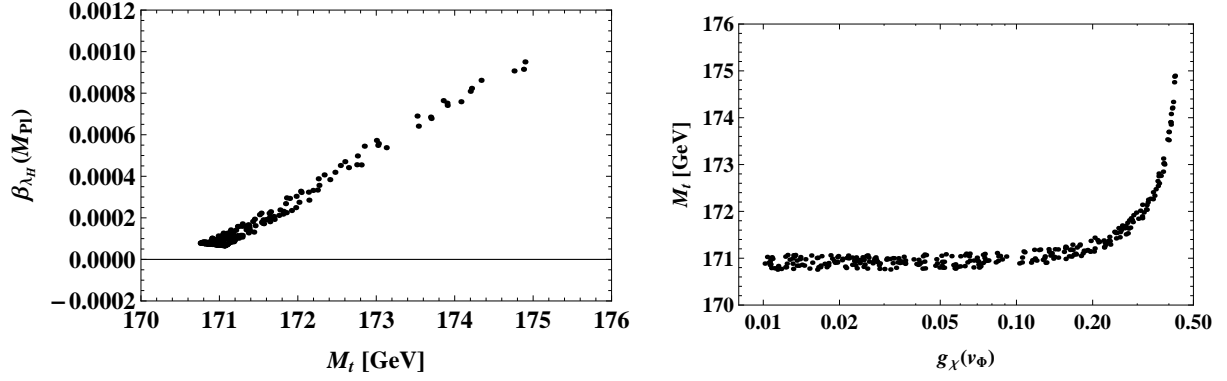


Figure 3: Left: Relation between the top pole mass and the β function of λ_H at the Planck scale. Right: g_χ dependences on the top pole mass. The dots realize the Higgs mass in a range of Eq. (1).

before λ_Φ becomes negative. Then, the β function of λ_Φ becomes

$$\beta_{\lambda_\Phi}(\mu < M_{Z'}, M_N) = \frac{1}{(4\pi^2)} [20\lambda_\Phi^2 + 2\lambda_{\text{mix}}^2], \quad (26)$$

up to the one-loop level. It does not include contributions of loop diagrams which have internal lines of the Z' boson and/or the right-handed neutrinos. Since both λ_Φ and λ_{mix} are numerically almost equal to zero around v_Φ , i.e., $\beta_{\lambda_\Phi}(\mu < M_{Z'}, M_N) \simeq 0$, it is reasonable to consider $\lambda_\Phi(\mu < M_{Z'}, M_N) \simeq \lambda_\Phi(M_{Z'}) \simeq \lambda_\Phi(M_N)$.² Thus, we can find that the parameter space of $g_\chi (\simeq y_M) \lesssim 0.055$ is excluded by $\lambda_\Phi(\mu < M_{Z'}, M_N) < 0$, which is shown as the left shaded region in Fig. 2. This constraint corresponds to $v_\Phi \lesssim 3.3 \times 10^5 \text{ GeV}$, $M_\Phi \gtrsim 2.8 \text{ GeV}$, $M_{Z'} \lesssim 3.7 \text{ TeV}$, and $M_N \lesssim 4.1 \text{ TeV}$, respectively.

As for $N_\nu = 2$ and 3, we find that $\lambda_\Phi > 0$ is not a constrained condition. For $N_\nu = 2$, we required that the running of λ_Φ is monotonically decreasing from the EW scale to the Planck scale, as in Fig. 1 (b). Since λ_Φ becomes rather larger at lower energy scales, λ_Φ is positive at any energy scale. Thus, the condition $\lambda_\Phi > 0$ gives no constraint for $N_\nu = 2$. For $N_\nu = 3$, the running of λ_Φ is the similar to that for $N_\nu = 1$, but the gradient of the running is much gentler, as in Fig. 1 (c). Then, even for $g_\chi \sim 0.01$ the Z' boson and the right-handed neutrinos are decoupled before λ_Φ becomes negative. Therefore, the small g_χ regions are almost not constrained for $N_\nu = 3$.

Next, we consider the mixing coupling between the scalar fields λ_{mix} . The vacuum stability requires $4\lambda_H\lambda_\Phi - \lambda_{\text{mix}}^2 > 0$, which means the large mixing can be excluded. When both λ_H and λ_Φ are positive, the inequality is almost always satisfied because of $\lambda_H \gg |\lambda_{\text{mix}}|$. On the other hand, the inequality cannot be explicitly satisfied when either λ_H or λ_Φ is negative. Then, we can find that the condition $4\lambda_H\lambda_\Phi - \lambda_{\text{mix}}^2 > 0$ is almost

² Here, we consider the tree-level matching condition, that is, the running couplings have no gaps at $M_{Z'}$ and M_N .

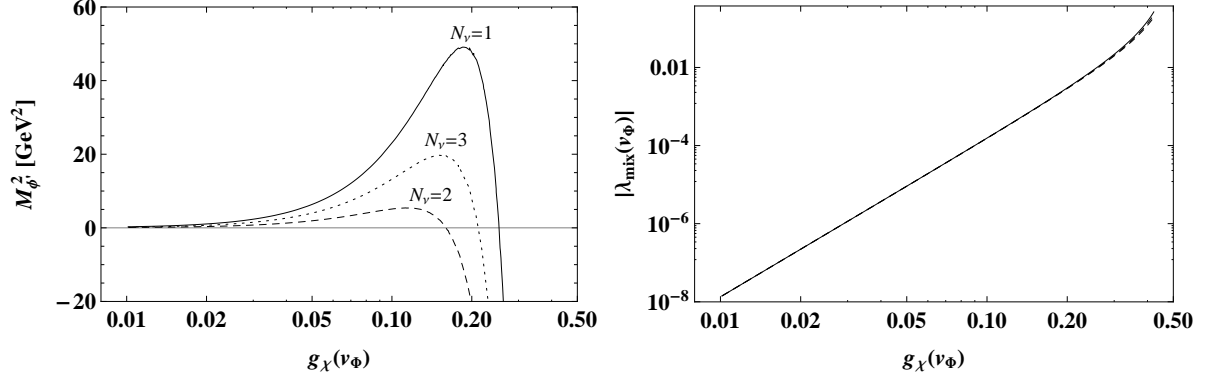


Figure 4: g_χ dependences on the lighter scalar mass squared eigenvalue (left) and the scalar mixing coupling (right). The solid, dashed, dotted lines correspond to $N_\nu = 1, 2$, and 3 respectively.

	$N_\nu = 1$	$N_\nu = 2$	$N_\nu = 3$
g_χ	$0.055 \lesssim g_\chi \lesssim 0.25$	$g_\chi \lesssim 0.16$	$g_\chi \lesssim 0.23$
v_Φ	$1.3 \text{ TeV} \lesssim v_\Phi \lesssim 3.3 \times 10^5 \text{ GeV}$	$3.8 \text{ TeV} \lesssim v_\Phi$	$2.0 \text{ TeV} \lesssim v_\Phi$
M_ϕ	$2.8 \text{ GeV} \lesssim M_\phi \lesssim 12 \text{ GeV}$	$M_\phi \lesssim 4.2 \text{ GeV}$	$M_\phi \lesssim 7.7 \text{ GeV}$
$M_{Z'}$	$650 \text{ GeV} \lesssim M_{Z'} \lesssim 3.7 \text{ TeV}$	$1.2 \text{ TeV} \lesssim M_{Z'}$	$860 \text{ GeV} \lesssim M_{Z'}$
M_N	$720 \text{ GeV} \lesssim M_N \lesssim 4.1 \text{ TeV}$	$1.1 \text{ TeV} \lesssim M_N$	$720 \text{ GeV} \lesssim M_N$

Table 2: Allowed parameter regions for the physical quantities.

the same as the condition $\lambda_H > 0$. Note that $4\lambda_H\lambda_\Phi - \lambda_{\text{mix}}^2 > 0$ cannot be satisfied in all energy scales, since $\lambda_H > 0$ cannot be satisfied below the Planck scale in order to realize the Higgs mass of 125 GeV as mentioned above. Thus, we try to constrain λ_{mix} in other conditions, that is, the positive definiteness of the scalar mass squared eigenvalues. The lighter scalar mass squared M_ϕ^2 given by Eq. (19) would be negative for a large $|\lambda_{\text{mix}}|$. The left figure of Fig. 4 shows that M_ϕ^2 becomes negative for the large g_χ region, which corresponds to a large mixing region (see the right figure). Since the running of λ_{mix} is almost the same for any $N_\nu = 1-3$, the relation between g_χ and λ_{mix} is also the same. Thus, considering the positive definiteness of the scalar mass squared eigenvalues, we can find that large g_χ regions are excluded in $g_\chi \gtrsim 0.25, 0.16$, and 0.23 for $N_\nu = 1, 2$, and 3, respectively. For example, in $N_\nu = 1$ case, it is shown as the right shaded region in Fig. 2. This constraint corresponds to $v_\Phi \gtrsim 1.3 \text{ TeV}$, $M_\phi \lesssim 12 \text{ GeV}$, $M_{Z'} \gtrsim 650 \text{ GeV}$, and $M_N \gtrsim 720 \text{ GeV}$, respectively. Therefore, the physical quantities are constrained from both above and below for $N_\nu = 1$. We show the allowed parameter regions for the physical quantities in Table 2. In fact, the ATLAS and CMS experiments have obtained larger lower bounds for $M_{Z'}$ than those in Table 2 as mentioned below.

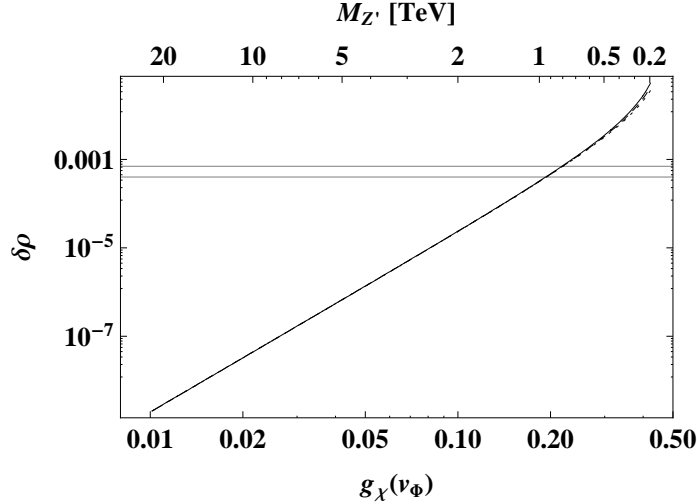


Figure 5: g_χ and $M_{Z'}$ dependence on $\delta\rho$. The lower and upper horizontal lines correspond to the central value and the upper bound at 1σ , respectively. The solid, dashed, dotted lines correspond to $N_\nu = 1, 2$, and 3 respectively.

4 Experimental bounds

In this section, we mention the experimental bounds. When there is gauge mixing between the Z and Z' bosons in the EW scale, it is dangerous since the ρ -parameter deviates from unity at the tree level. Let us estimate the deviation of the ρ -parameter [31]. The tree-level ρ -parameter is defined by $\rho_0 = M_W^2/(M_1^2 c_W^2)$, where $M_W^2 = g_2^2 v_H^2/4$ is the W boson mass, and $c_W^2 = g_2^2/\sqrt{g_Y^2 + g_2^2}$ is the Weinberg angle. The deviation of the ρ -parameter $\delta\rho \equiv \rho_0 - 1$ is always positive because of $M_1 < M_Z$. From Eq. (23), $\delta\rho$ is approximately given by

$$\delta\rho \equiv \rho_0 - 1 \approx \frac{v_H^2}{4 \left[\left(M_{Z'}^2 + \frac{1}{4} (g_{\text{mix}} - \frac{4}{5} g_\chi)^2 v_H^2 \right) - M_Z^2 \right]} \left(g_{\text{mix}} - \frac{4}{5} g_\chi \right)^2. \quad (27)$$

We can find that $\delta\rho$ is proportional to $\tan 2\theta_Z$. Thus, $\delta\rho$ is vanishing in the limit of $\tan 2\theta_Z \rightarrow 0$, which is necessarily required.

Now, we can compare $\delta\rho$ with its experimental bound $\rho_0 = 1.0004^{+0.0003}_{-0.0004}$ [40]. Figure 5 shows g_χ and $M_{Z'}$ dependence on $\delta\rho$, in which the lower and upper horizontal lines correspond to the central value and the upper bound at 1σ , respectively. We can see that $\delta\rho$ is almost independent of N_ν , since N_ν does not change the running of gauge couplings up to one-loop level. $\delta\rho$ becomes larger as g_χ becomes larger, equivalently $M_{Z'}$ becomes lower. Then, the central value of ρ_0 and its upper bound at 1σ correspond to $g_\chi \simeq 0.19$ and 0.21 , equivalently $M_{Z'} \simeq 950$ GeV and 820 GeV, respectively. Thus, $M_{Z'}$ should be heavier than 820 GeV.

Finally, we mention the Z' boson mass bounds obtained by the recent collider experiments (see Ref. [41] for a review). Currently, the highest mass bounds on the Z' boson are obtained by searches at the LHC by the ATLAS and CMS experiments. The most recent results are based on the search for the heavy neutral gauge boson decaying to e^+e^- or $\mu^+\mu^-$ pairs. The ATLAS obtains the exclusion limits at 95% C.L. as $M_{Z'} > 2.24 \text{ TeV}$ for the $U(1)_\chi$ model. It is used the center-of-mass energy $\sqrt{s} = 8 \text{ TeV}$ pp collision data set collected in 2012 corresponding to an integrated luminosity of approximately $5.9 (e^+e^-) / 6.1 (\mu^+\mu^-) \text{ fb}^{-1}$ [42]. Similarly, the CMS obtains the exclusion limits at 95% C.L. as $M_{Z'} > 2.59 \text{ TeV}$ for the sequential standard model with SM-like couplings [43]. It used the $\sqrt{s} = 8 \text{ TeV}$ pp collision data set and $\sqrt{s} = 7 \text{ TeV}$ data set collected by the CMS experiment in 2011 corresponding to an integrated luminosities of up to 4.1 fb^{-1} [44].

In addition, another constraint is obtained by measurements of $e^+e^- \rightarrow f\bar{f}$ above the Z -pole at the LEP-II, where f denotes various SM fermions. When $M_{Z'}$ is larger than the largest collider energy of the LEP-II, which is about 209 GeV , one can effectively perform an expansion in $s/M_{Z'}^2$ for four fermion-interactions. Then, effective four-fermion interactions have been bounded by the LEP-II. Since the amplitudes of the Z' boson mediating interactions are proportional to $g_{Z'}^2/M_{Z'}^2$, the bound can be obtained as the ratio $M_{Z'}/g_{Z'}$, where $g_{Z'}$ is a flavor independent Z' gauge coupling. Using the single channel estimation, one can obtain the lower bound $M_{Z'}/g_\chi \gtrsim 3.8 \text{ TeV}$ for the $U(1)_\chi$ model [45]. In a recent parameter fitting analysis, the lower bound $M_{Z'}/g_\chi \geq 4.8 \text{ TeV}$ has been obtained at 99% C.L. [46].

Let us summarize all the constraints in Fig. 6. In the flatland scenario, the physical quantities are uniquely determined once one parameter is fixed. The relation between $M_{Z'}$ and g_χ are given by the black solid line. The shaded regions show constraints obtained by Sects. 3 and 4. The constraint from $\lambda_\Phi < 0$ is obtained only in the $N_\nu = 1$ case, while $\lambda_\Phi < 0$ gives no constraints in the $N_\nu = 2$ and 3 cases. Thus, the constraints for $N_\nu = 2$ and 3 are the same as obtained by the LHC experiments: $2.24 (2.59) \text{ TeV} \lesssim M_{Z'}$, where the lower bound corresponds to the ATLAS (CMS) result. On the other hand, we can find that the Z' boson mass for $N_\nu = 1$ is tightly restricted: $2.24 (2.59) \text{ TeV} \lesssim M_{Z'} \lesssim 3.7 \text{ TeV}$, where the upper bound is obtained by the condition of $\lambda_\Phi > 0$.

5 Conclusion

We have studied the scale invariant local $U(1)_\chi$ model with vanishing scalar potential at the Planck scale, which is the so-called flatland scenario. The $U(1)_\chi$ symmetry is broken by the CW mechanism, and it subsequently leads to EW symmetry breaking. Using the conditions for the CW mechanism to successfully occur and realize $M_h \simeq 125 \text{ GeV}$ and

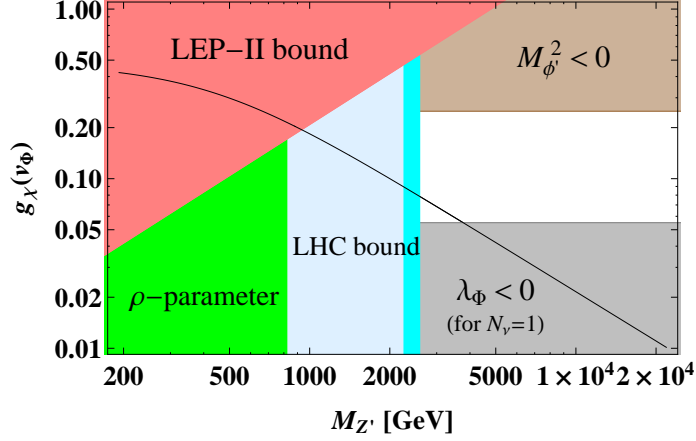


Figure 6: All the constraints on $M_{Z'}$ and g_{χ} . The black line corresponds to the flatland prediction for $N_{\nu} = 1$. The shaded regions show constraints obtained by Sect. 3 and 4.

$v_H \simeq 246$ GeV, the physical quantities are uniquely determined once one parameter is fixed.

To constrain the physical quantities, we have investigated the vacuum stability using the two-loop RGEs. First, we have considered $\lambda_H > 0$ at all energy scales, and found that it is impossible to realize $M_h \simeq 125$ GeV while keeping $\lambda_H > 0$, the same situation as in the SM. In the following results, we have given up $\lambda_H > 0$ at any energy scale.

Next, we have considered $\lambda_{\Phi} > 0$ at all energy scales. When the number of relevant Majorana Yukawa couplings of the right-handed neutrinos is one, i.e., $N_{\nu} = 1$, the lower bound of the $U(1)_{\chi}$ gauge coupling g_{χ} has been obtained by considering the decoupling effects of the Z' boson and the right-handed neutrinos. In practice, the condition $\lambda_{\Phi} > 0$ is reasonable to consider $\lambda_{\Phi}(\mu < M_{Z'}, M_N) \simeq \lambda_{\Phi}(M_{Z'}) \simeq \lambda_{\Phi}(M_N) > 0$ because of $\beta_{\lambda_{\Phi}}(\mu < M_{Z'}, M_N) \simeq 0$. Then, we have found the lower bound of g_{χ} , shown as the left shaded region in Fig. 2. However, the condition $\lambda_{\Phi} > 0$ does not constrain in the $N_{\nu} = 2$ and 3 cases. For $N_{\nu} = 2$, the running of λ_{Φ} is monotonically and slowly decreasing from the EW scale to the Planck scale, quite untypically. Thus, the condition $\lambda_{\Phi} > 0$ gives no constraint in the $N_{\nu} = 2$ case, since λ_{Φ} is always positive. For $N_{\nu} = 3$, the running of λ_{Φ} is similar to that for $N_{\nu} = 1$, but the gradient of the running is much gentler. Then, the Z' boson and the right-handed neutrinos are decoupled before λ_{Φ} becomes negative even for $g_{\chi} \sim 0.01$. Therefore, the small g_{χ} regions are almost not constrained in the $N_{\nu} = 3$ case.

In addition, we have discussed the positive definiteness of the scalar mass squared eigenvalues. The large g_{χ} generates the large scalar mixing, and it would make the lighter mass squared eigenvalue be negative. Thus, it gives the upper bound of g_{χ} , which is

shown as the right shaded region in Fig. 2. As a result, considering the vacuum stability and the positive definiteness of the scalar mass squared eigenvalues, we have found the allowed parameter regions for the physical quantities as in Table 2.

Finally, we have mentioned the experimental bounds on $M_{Z'}$. To obtain the constraints on $M_{Z'}$, we have discussed the following experiments: the deviation of the ρ -parameter from unity, the pp collision to e^+e^- or $\mu^+\mu^-$ at the LHC, and $e^+e^- \rightarrow f\bar{f}$ at the LEP-II. As a result, we have obtained the constraints shown in Fig. 6, and found that the Z' boson mass for $N_\nu = 1$ is tightly restricted to $2.24 \text{ (2.59) TeV} \lesssim M_{Z'} \lesssim 3.7 \text{ TeV}$, where the lower bound corresponds to the ATLAS (CMS) result.

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Appendix

β functions in the $U(1)_\chi$ extended SM

The RGE of coupling x is given by $dx/d\ln\mu = \beta_x$, in which μ is a renormalization scale.

The β functions in the $U(1)_\chi$ extended SM are given by

$$\beta_{g_Y} = \frac{g_Y^3}{(4\pi)^2} \left[\frac{41}{6} \right], \quad \beta_{g_2} = \frac{g_2^3}{(4\pi)^2} \left[-\frac{19}{6} \right], \quad \beta_{g_3} = \frac{g_3^3}{(4\pi)^2} [-7], \quad (28)$$

$$\beta_{g_\chi} = \frac{g_\chi}{(4\pi)^2} \left[\frac{196}{25} g_\chi^2 + \frac{41}{6} g_{\text{mix}}^2 - \frac{4}{15} g_{\text{mix}} g_\chi \right], \quad (29)$$

$$\beta_{g_{\text{mix}}} = \frac{1}{(4\pi)^2} \left[g_{\text{mix}} \left(\frac{41}{6} (2g_Y^2 + g_{\text{mix}}^2) + \frac{196}{25} g_\chi^2 \right) - \frac{4}{15} g_\chi (g_Y^2 + g_{\text{mix}}^2) \right], \quad (30)$$

$$\beta_{y_t} = \frac{y_t}{(4\pi)^2} \left[\frac{9}{2} y_t^2 - 8g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} (g_Y^2 + g_{\text{mix}}^2) - \frac{6}{25} g_\chi^2 + \frac{3}{5} g_{\text{mix}} g_\chi \right], \quad (31)$$

$$\beta_{y_{M_i}} = \frac{y_{M_i}}{(4\pi)^2} [4y_{M_i}^2 + 2\text{Tr}(Y_M^2) - 6g_\chi^2], \quad (32)$$

$$\begin{aligned} \beta_{\lambda_H} = & \frac{1}{(4\pi)^2} \left[\lambda_H \left(24\lambda_H + 12y_t^2 - 3(g_Y^2 + g_{\text{mix}}^2) - 9g_2^2 - \frac{48}{25} g_\chi^2 + \frac{24}{5} g_{\text{mix}} g_\chi \right) \right. \\ & \left. + \lambda_{\text{mix}}^2 - 6y_t^4 + \frac{3}{8} \left(2g_2^4 + \left\{ g_2^2 + g_Y^2 + \left(g_{\text{mix}} - \frac{4}{5} g_\chi \right)^2 \right\}^2 \right) \right], \end{aligned} \quad (33)$$

$$\beta_{\lambda_\Phi} = \frac{1}{(4\pi)^2} [\lambda_\Phi (20\lambda_\Phi + 8\text{Tr}(Y_M^2) - 48g_\chi^2) + 2\lambda_{\text{mix}}^2 - 16\text{Tr}(Y_M^4) + 96g_\chi^4], \quad (34)$$

$$\begin{aligned} \beta_{\lambda_{\text{mix}}} = & \frac{1}{(4\pi)^2} \left[\lambda_{\text{mix}} \left(12\lambda_H + 8\lambda_\Phi + 4\lambda_{\text{mix}} + 6y_t^2 + 4\text{Tr}(Y_M^2) - 24g_\chi^2 \right) \right. \\ & \left. - \frac{3}{2} \left\{ 3g_2^2 + g_Y^2 + \left(g_{\text{mix}} - \frac{4}{5} g_\chi \right)^2 \right\} \right] + 12 \left(g_{\text{mix}} - \frac{4}{5} g_\chi \right)^2 g_\chi^2, \end{aligned} \quad (35)$$

up to the one-loop level. We have only included the top quark Yukawa coupling, and omitted the other Yukawa couplings of the SM particles, since they do not contribute significantly to the Higgs quartic coupling and gauge couplings. In this paper, we have used two-loop β functions, which are obtained by SARAH [47].

To solve the RGEs, we take the following boundary conditions [2]:

$$g_Y(M_t) = 0.35761 + 0.00011 \left(\frac{M_t}{\text{GeV}} - 173.10 \right), \quad (36)$$

$$g_2(M_t) = 0.64822 + 0.00004 \left(\frac{M_t}{\text{GeV}} - 173.10 \right), \quad (37)$$

$$g_3(M_t) = 1.1666 - 0.00046 \left(\frac{M_t}{\text{GeV}} - 173.10 \right) + 0.00314 \left(\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \right), \quad (38)$$

$$y_t(M_t) = 0.93558 + 0.00550 \left(\frac{M_t}{\text{GeV}} - 173.10 \right) - 0.00042 \left(\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \right) \quad (39)$$

$$\alpha_3(M_Z) = 0.1184 \pm 0.0007, \quad (40)$$

where M_t is the pole mass of top quark.

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